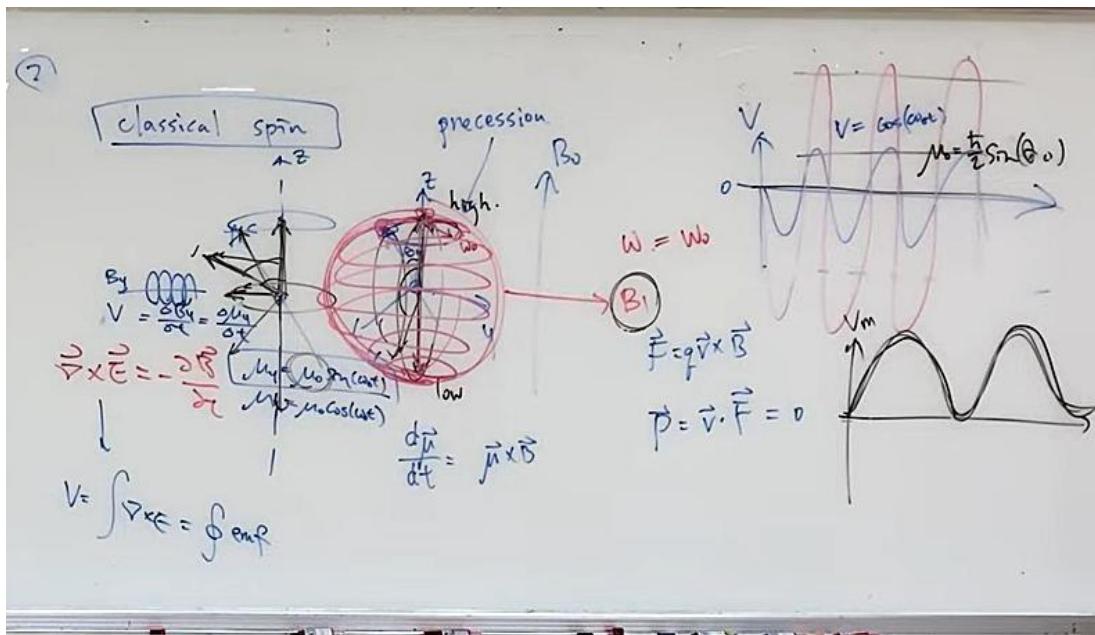
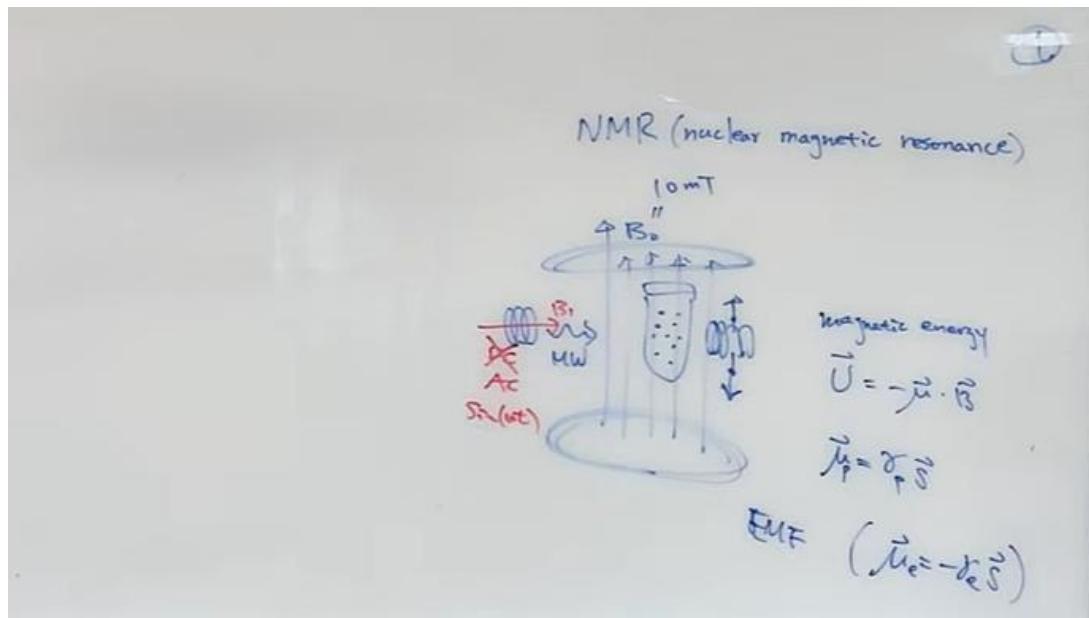




# Lecture9 Classical and quantum spin





Quantum Optics

$$|\psi\rangle = \frac{\cos\theta e^{i\phi}}{C_g} |g\rangle + \frac{\sin\theta e^{i\phi}}{C_e} |e\rangle$$

$$|C_g|^2 + |C_e|^2 = 1$$

$$C_g = \cos\theta e^{i\phi}$$

$$C_e = \sin\theta e^{i\phi} e^{i\phi} e^{-i(\phi_e - \phi_g)}$$

$$P = |\psi|^2 = \langle \psi | \psi \rangle$$

$$e^{i\phi(\psi)} = |C_g|^2 + |C_e|^2$$

$$P_g = |C_g|^2$$

$$P_e = |C_e|^2$$

**Block equation**

$$\frac{d\vec{B}}{dt} = \vec{P} \times \vec{B}$$

$$\rightarrow \frac{d\vec{B}}{dt} = Y_p \vec{S} \times \vec{B} = Y_p \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ S_x & S_y & S_z \\ B_x & B_y & B_z \end{vmatrix} \quad \text{optical Bloch equation}$$

$$\rightarrow \frac{dS_x}{dt} = Y_p (B_z S_y - B_y S_z) \quad \frac{dS_y}{dt} = Y_p (B_x S_z - B_z S_x) \quad \frac{dS_z}{dt} = Y_p (B_y S_x - B_x S_y)$$

$$\frac{d\vec{v}}{dt} = -\Delta \vec{v} \quad (x, 0, -z)$$

$$\frac{dv}{dt} = \chi w + \Delta u$$

$$\frac{dw}{dt} = -\chi v$$

$$\Delta = \omega - \omega_0$$

$$C_g(t) = e^{-\frac{i\Delta t}{\hbar}} \left\{ \left[ \cos\left(\frac{\Delta t}{\hbar}\right) + \frac{i\Delta}{\hbar} \sin\left(\frac{\Delta t}{\hbar}\right) \right] C_g(0) + \frac{i\chi}{\hbar} e^{-i\phi} \sin\left(\frac{\Delta t}{\hbar}\right) C_e(0) \right\}$$

$$C_e(t) = e^{-\frac{i\Delta t}{\hbar}} \left\{ \left[ \frac{i\chi}{\hbar} e^{i\phi} \sin\left(\frac{\Delta t}{\hbar}\right) C_g(0) + \left[ \cos\left(\frac{\Delta t}{\hbar}\right) - \frac{i\Delta}{\hbar} \sin\left(\frac{\Delta t}{\hbar}\right) \right] C_e(0) \right\}$$

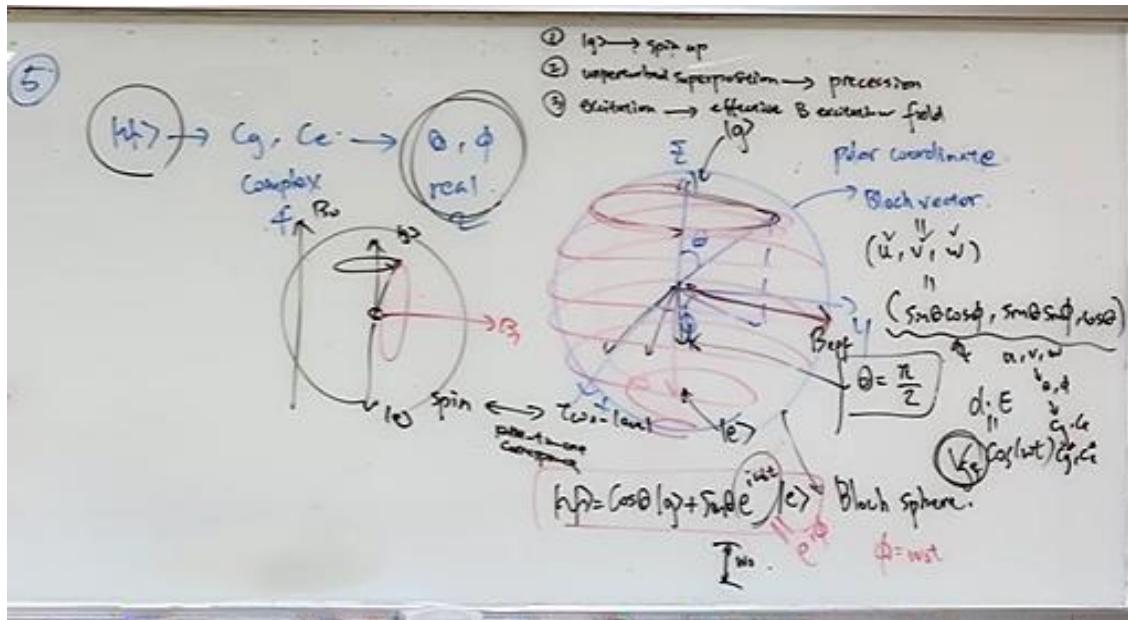
$$|\psi\rangle = C_g |g\rangle + C_e |e\rangle$$

$$P_e = |C_e|^2 = \left| \overbrace{C_g(0)}^0 + \overbrace{\Delta C_e(0)}^{\Delta} \right|^2 = \overbrace{C_g(0)}^0 + \overbrace{\Delta |C_e(0)|^2}^{\Delta} + \overbrace{\Delta C_g(0) C_e(0)}^{\Delta C_g C_e}$$

$$P_g = |C_g|^2$$

$$C_S = \langle \psi | H | \psi \rangle = (C_g^* C_g + C_e^* C_e) = C_g^* C_g + (C_g^* C_e + C_e^* C_g)$$





Angular motion

$$\rho = |A| > \langle A \rangle H = (C_g, C_e) \otimes \begin{pmatrix} g \\ e \end{pmatrix} \quad \leftarrow \text{torque}$$

$$\langle A \rangle H = C_g \langle g \rangle + C_e \langle e \rangle$$

$$\frac{dC_g}{dt} = \frac{dC_g}{dt} + \frac{dC_e}{dt} C_g = (C_g \dot{v}) + (C_e \dot{v})$$

$$\frac{dC_e}{dt} = \frac{1+w}{1-w} C_g \dot{v} + \frac{1-w}{1+w} C_e \dot{v}$$

$$\frac{dC_g}{dt} = \frac{1+w}{1-w} C_g \dot{v} + \frac{1-w}{1+w} C_e \dot{v}$$

$$\frac{dC_e}{dt} = \frac{1-w}{1+w} C_g \dot{v} + \frac{1+w}{1-w} C_e \dot{v}$$

$$(C_g^2 + C_e^2 = 1)$$

$$C_g = \cos\left(\frac{\theta}{2}\right) e^{i\phi} \quad C_e = \sin\left(\frac{\theta}{2}\right) e^{i\phi} e^{-i\phi}$$

$$P = |A|^2 = \langle A \rangle H$$

$$e^{i(\omega t + \phi)} = |A|^2 = (C_g^2 + C_e^2)^2 = 1$$

$$P_{\text{optical}} = \frac{1}{2} \rho^2 = \frac{1}{2} (C_g^2 + C_e^2)^2 = \frac{1}{2} (C_g^2 + C_e^2) = \frac{1}{2} (1 + \cos^2 \theta)$$





- ✓ - classical spin (NMR)
- ✓ - quantum spin (Bloch sphere)
- Hahn spin echo

